PH4204: High Energy Physics

1. **Model of the Charged Current:** Consider the processes $e + \nu_e \to e + \nu_e$ and $e + \bar{\nu}_e \to e + \bar{\nu}_e$.
   (a) Assuming the matrix elements of the above processes of the form $\mathcal{M} = (G/\sqrt{2}) J^\alpha(e)^\dagger J_\alpha(e)$ calculate their total cross-sections. Take all fermions to be mass-less.
   (b) The above two cross-sections diverge quadratically with E_{cm}. To controll the behaviour, propose a massive charged spin one boson with appropriate couplings and propators and re-calculate the cross-sections. (*Hint:* See next problem.)

2. **Intermediate Vector Boson theory:** The weak force is postulated to be carried by the exchange of some charged *intermediate vector bosons*, $W^\pm$ with coupling to fermions as (for charged currents)

$$\frac{g}{2\sqrt{2}} \; \bar{\Psi}_i \gamma^\mu (1-\gamma^5) \Psi_j W^\pm_\mu \qquad\qquad (i,j) = (e, \nu_e)$$

   The polarization sum for $W^\pm$ is given by

$$\sum_\lambda \epsilon_\mu(k) \epsilon_\nu(k)^* = -\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2} \right).$$

   (a) Draw the Feynman diagrams for $e^+ + e^- \to W^+ + W^-$ using $e^\pm$, $W^\pm$ and neutrinos only.
   (b) Calculate the matrix element square for above process. Comment in its dependence on E_{cm}. Electrons and neutrinos to be treated as massless, but not W-boson.
   (c) Calculate the total cross-section for the above process (i.e. integrated over scattering angle) and comment on its dependence on E_{cm}.
   (d) If the $k_\mu k_\nu / m^2$ term is dropped from the expression for polarization sum, how the E_{cm} dependence modified.
   (*Hint: You may use the FORM program to calculate the trace.*).

3. **Adding neutral current:** In the above theory, we add a neutral *intermediate vector boson*, W^0, with coupling

$$\frac{g}{2\sqrt{2}} \; \bar{\Psi}_i \gamma^\mu (1-\gamma^5) \Psi_j W^0_\mu \qquad\qquad i = j$$

   and same mass and polarization sum as given above for $W^\pm$. The interaction between the charged and neutral current is given by the vertex

$$W^+_\mu(p) W^-_\nu(q) W^0_\rho(r) : \quad \alpha \Big[(p-q)_\rho g_{\mu\nu} + (q-r)_\mu g_{\nu\rho} + (r-p)_\nu g_{\mu\rho} \Big]$$

   (a) Draw the additional digrams for the $e^+ + e^- \to W^+ + W^-$ process.
   (b) Calculate the total matrix element square for this process, including both the diagrams.
   (c) Comment of the possible value of the coupling α that one requires to cancel the bad high energy behaviour of the total matrix element square.
   (*Hint: You may use the FORM program to calculate the trace.*).

4. **Interacting scalar and fermions:** Consider the field theory of three interacting scalars and fermions

$$\begin{aligned}
\mathcal{L}_2 \;=\; & \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) - \frac{m_1^2}{2}\phi_1^2 - \frac{\lambda_1}{4!}\phi_1^4 \quad + \quad \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) - \frac{m_2^2}{2}\phi_2^2 - \frac{\lambda_2}{4!}\phi_2^4 \\
+ \; & \bar{\psi}[i\partial_\mu \gamma^\mu - m]\psi + \; g_s \, \bar{\psi}\psi \, \phi_1 + \; i g_p \, \bar{\psi}\gamma^5\psi \, \phi_2
\end{aligned}$$

(a) Write down the free Fenyman propagator for two scalar fields, ϕ_1, ϕ_2 and fermion field ψ.

(b) Write down the Feynman rules for this theory.

(c) Consider the two point correlation $\langle\Omega|\mathcal{T}\{\phi_1(x)\phi_2(y)\}|\Omega\rangle$. It is explicitly zero at tree level. Draw the Feynman diagram for it at one loop, write the expressions and show that it is zero at one loop as well.

5. **Constructing a $U(1)$ Gauge theory:** We try to construct a gauge theory based on $U(1)$ gauge symmetry that has many features of the Standard model.

   (a) *Guage transformations $U(1)$:* Assume two left chiral fermions, ψ_1^L and ψ_2^L, with charges $Q_1^L = Q_2^L = +1$; two right chiral fermions ψ_1^R and ψ_2^R, with charges $Q_1^R = Q_2^R = 0$; one charged scalar Φ with charge $Q_s = +1$ under a $U(1)$ guage group. List the gauge transformations of these five fields.

   (b) *Covariant derivatives and Lagrangian:* Write down the expression for the covariant derivatives for above five fields under $U(1)$ gauge. Also write down the $U(1)$ gauge invariant Lagrangian for these five fields and the gauge fields. Assume them to be mass-less.

   (c) *Higgs mechanism for $U(1)$ gauge:* Add suitable terms in the above obtained Lagrangian such that the scalar Φ gets a vacuum expectation value (VEV) denote by v and the $U(1)$ gauge boson gets a mass. Write down expressions of masses for the $U(1)$ gauge bosons and the Higgs boson.

   (d) *Yukawa interaction:* Propose a suitable Yukawa interaction between scalar Φ and fermions ψ_i^L, ψ_i^R that is gauge invariant and gives a mass to both the fermions after Φ gets a VEV.

   (e) *Mass mixing:* Assuming there is a Yukawa interaction between ψ_1^L and ψ_2^R etc., write down the complete Lagrangian from above.